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DIFFERENT PARTNERS-DIFFERENT PLACES

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This is the second of two papers we have written on this subject. The first paper is a careful mathematical statement of how we figured it all out. This second paper tries to motivate the key ideas from the first paper without being too rigorous about it. It is addressed to people with some mathematical knowledge (but not too much) and an interest in dancing. Throughout this paper we offer various "Assertions" which will be relatively easy to understand as stated, although it may not always be obvious why they are true. The backup support is, of course, in the first paper. We will be happy to explain things further for anyone who is interested.

The Problem

Four-couple dances usually have certain characteristics in common:

1. The dance is done 4 times through.
2. Each time through the dance, the dancers are rearranged in some way:
   (a) Some or all dancers move to *new places* in the set, and/or
   (b) Some or all dancers are paired with *new partners* each time.
3. The *same* sequence of figures is done each time through.
4. When the dance ends, everyone is back home with original partner.

There is one property that, *by its absence,* also seems to be common to four-couple dances: There do not seem to be any four-couple dances in which every dancer dances in a *different place* with a *different partner* each time through the dance. The rearrangements that *do* occur typically fall into one of three categories:

1. *Same Partners–Different Places*: Each *couple* dances the figures once from each of the 4 positions.

2. *Different Partners–Some Different Places*: The men "stay home" as the women travel through the 4 positions; or the opposite–the women stay home and men travel.

3. *Some Different Partners–Different Places*: Everyone gets to all four places during the dance but encounters original partner at least once and other partners the remaining times through.

A fourth possible category in this list seems to be empty, at least as far as the authors can determine:

4. *Different Partners–Different Places* (**DP/DP**): Every dancer is in a *different place* with a *different partner* every time through the dance. No one has the same partner twice during the dance, and no one begins the set of figures from the same place twice during the dance.

The Paper

In this paper (and more formally in the first paper) we address two questions:

1. *Is it possible to create a four-couple DP/DP dance?*

The answer, as we shall see, is *Yes*, it is possible, although the ways to do it are limited.

2. *Given that it is possible, how exactly is it done?*

We will summarize some rules for getting the job done.

We will address these two questions in the next two sections. Then we will offer a few comments about what is required to write *good* DP/DP dances. We need to acknowledge at the outset that what is *possible* in dance writing may not be *desirable*. The choreographer, after all, wants to write DP/DP dances that dancers will actually enjoy. In the last section, we show an example of what can be done.

We will analyze the problem by separating the *different places* question from the *different partners* question and examining first one and then the other. Then we will bring them back together.

DIFFERENT PLACES

In this section, we will not worry about matching people with partners and concentrate instead just on how to get four people, the men or the women, to four places in four moves.

We will have little to say here about the actual *figures* that the dance choreographer puts together in a DP/DP dance (swing partner, do-si-do, circle left, etc.). That creative process remains for the choreographer to face. Rather we will be working in terms of *progressions*.

Definition: A ***progression*** describes the *net effect of doing a set of figures.* It summarizes what happens to four dancers during a single time through the dance. A complete dance is comprised of four progressions.[1]

There are many ways to move dancers around the set but only a limited number qualify as *real* progressions for our purposes.

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[1] This definition is slightly different from the one used in our first paper where "progression" is used to describe what happens to *all eight* dancers.

Assertion 1: In four-couple dances, if four dancers (say, the men) are to get to all four places in four times through the dance, then they all have to move to a different place every time through. No one can stay in the same place twice in a row or go back to a place they have already been.

Assertion 2: There are only nine ways to move dancers around that can qualify as real progressions.

The nine ways fall into two categories, which we will call *cycles* and *exchanges*. Here is an example of each type:

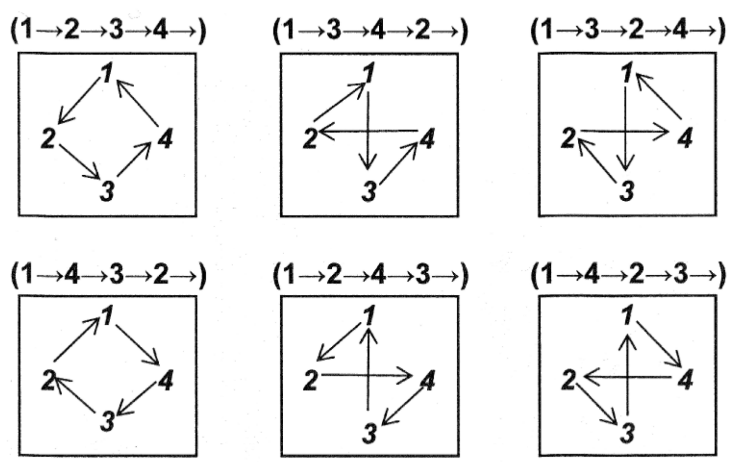
Cycle Progression:
The person in place 1 goes to place 2, the person in place 2 goes to place 3, the person in place 3 go to place 4, and the person in place 4 goes to place 1. This is pretty wordy, so we will abbreviate it as

(1→2→3→4→)

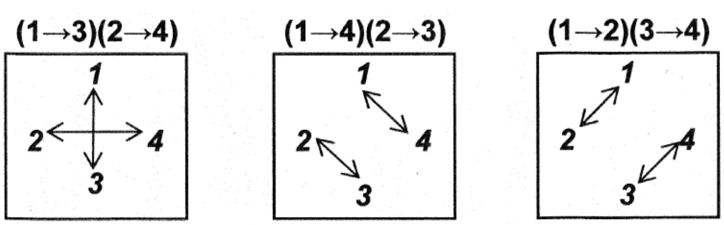
Exchange Progression:
People in places 1 and 2 switch places, and people in places 3 and 4 switch places, which can be abbreviated as

(1↔2) (3 ↔4)

There are six *cycle* progressions in all (here we are applying the moves to a square):



There are three *exchange* progressions:



Compatible (and Incompatible) Sequences

Having identified our candidate progressions, our next task is to figure out how to put together four of them in a sequence–either for the men or for the women-to satisfy the *different places* requirement. Each dancer must get to each of the four places *and home again* in four progressions. This is a little harder to do that it might sound because of the following:

> **Assertion 3:** Most combinations of two progressions in a row yield rearrangements that take some dancers back to places they have already visited.

In other words, *even though the individual progression move everyone to new places*, the net effect of two of them may not. Here is one example of the kind of problem that can arise:

> *The Pit Fall:* Suppose the first two progressions for the men in a four-couple longways dance are[2]

Begin	1st Progression	After 1st	2nd Progression	After 2nd
M1 M2 M3 M4	Bottom man go to the top; everyone else move down. (1→2→3→4→)	M4 M1 M2 M3	Middle men move to the nearest end; Men on the ends meet in middle and change places. (1→3→4→2→)	M1 M3 M4 M2

After these two progressions, everyone is in a new place, *except* the man who started in place 1. That man is back home after the two progressions. This cannot be allowed if that person is to get to all four places in four moves.

> **Definition:** A sequence of four progressions that avoids this "doubling back" problem (i.e., everyone gets to each of the four places *and home again* in four progressions) will be called a ***compatible sequence of progressions***.

Simple Compatible Sequences

Before discussing how to construct compatible sequences, let's jump ahead and look at a few. Probably the easiest way to create a compatible sequence is to choose one of the cycle progressions and do it four times in a row. In fact, this is the way that most four-couple dances are organized–everyone does the same sequence of figures (i.e., progressions) every time and ends up at home.

Suppose, for example, we choose (1→2→3→4→). Doing this progression four times meets the different places requirement perfectly. Table 1 shows how four women would move through a four-couple *longways* set. (This would apply as well in a square.) The top row shows the progressions they are using (same one every time), and the remaining rows show where

[2] Remember that the numbers in a progression are *place* numbers, not *people* numbers.

everyone is located after each time through. Basically, the woman at the bottom goes to the top of the set each time through.

Progressions⇒		(1→2→3→4→)	(1→2→3→4→)	(1→2→3→4→)	(1→2→3→4→)
Places	♬ **Home**	➘ ♬ ➚ **Arrangement 1**	➘ ♬ ➚ **Arrangement 2**	➘ ♬ ➚ **Arrangement 3**	**(Home ♬ Again) Arrangement 4**
1	W1	W4	W3	W2	W1
2	W2	W1	W4	W3	W2
3	W3	W2	W1	W4	W3
4	W4	W3	W2	W1	W4

Table 1

There are six cycle progressions, so that there are six *simple* compatible sequences of progressions. They all say "Do the same thing all four times."

Mixed Compatible Sequences

In addition to the simple compatible sequences, there are some less obvious compatible sequences that involve *mixing progressions together.* One must be careful about how the mixing is done, but just to demonstrate that it *is* possible, look at Table 2 which shows an alternative way the women might progress through a four-couple longways dance.

Progressions⇒		(1→2→3→4→)	(1↔3)(2↔4)	(1→4→3→2→)	(1↔3)(2↔4)
Places	♬ **Home**	➘ ♬ ➚ **Arrangement 1**	➘ ♬ ➚ **Arrangement 2**	➘ ♬ ➚ **Arrangement 3**	**(Home ♬ Again) Arrangement 4**
1	W1	W4	W2	W3	W1
2	W2	W1	W3	W4	W2
3	W3	W2	W4	W1	W3
4	W4	W3	W1	W2	W4

Table 2

The top row again shows four progressions, but they change each time through. In words, here is what the progressions say:

1^{st} Time: Person in *bottom* place moves to the top, everyone else moves down.
2^{nd} Time: People in 1^{st} and 3^{rd} places swap; people in 2^{nd} and 4^{th} places swap.
3^{rd} Time: Person in the *top* place moves to the bottom, everyone else moves up.
4^{th} Time: People in 1^{st} and 3^{rd} places swap; people in 2^{nd} and 4^{th} places swap.

One may verify from the table that this plan does in fact put all four women in all four places in four moves. Consequently, we have identified another compatible sequence.

Here is one more example of a compatible sequence. It uses a mix of exchange progressions.

Progressions⇒	(1↔2)(3↔4)	(1↔4)(2↔3)	(1↔2)(3↔4)	(1↔4)(2↔3)	
Places	♬ **Home**	➘ ♬ ➚ **Arrangement 1**	➘ ♬ ➚ **Arrangement 2**	➘ ♬ ➚ **Arrangement 3**	**(Home ♬ Again) Arrangement 4**
1	M1	M2	M3	M4	M1
2	M2	M1	M4	M3	M2
3	M3	M4	M1	M2	M3
4	M4	M3	M2	M1	M4

Table 3

The description in words in this case is different (and a little easier to remember):

1st Time: People in 1st and 2nd places swap; people in 3rd and 4th places swap.
2nd Time: People in 1st and 4th places swap; people in 2nd and 3rd places swap.
3rd Time: People in 1st and 2nd places swap; people in 3rd and 4th places swap.
4th Time: People in 1st and 4th places swap; people in 2nd and 3rd places swap.

Notice that the 1st and 3rd progressions are the same (same set of figures), and the 2nd and 4th are the same, so that the dancers' mental burden is not quite so heavy.

Constructing Compatible Sequences

How can we know which progressions work together well (as the ones above did) and which do not? There are two issues to worry about: people have to keep moving to new places and they have to get to all four places in four moves.

The answers to these questions lie in the following three assertions (based on Lemma 1 and Theorem 1 in our first paper): [3]

Assertion 4: The nine progressions can be organized into four little sets, which we will call *groups*, that guide us in knowing what works well with what. Here are the groups:[4]

[3] The next several assertions may feel as if they are just "coming out of blue"–they are understandable, but how do we know they are true? In fact, everything is nailed down properly in our first paper.

[4] Those familiar with mathematical Group Theory will recognize that, with the addition of an identity element to each one, these four sets are *real* subgroups of the symmetric group S_4.

I	II	III	IV
(1↔3)(2↔4)	(1→2→3→4→)	(1→3→4→2→)	(1→3→2→4→)
(1↔4)(2↔3)	(1→4→3→2→)	(1→2→4→3→)	(1→4→2→3→)
(1↔2)(3↔4)	(1↔3)(2↔4)	(1↔4)(2↔3)	(1↔2)(3↔4)

Observe that the exchange progressions constitute a group by themselves (Group I) and each of the exchange progressions also belongs to one of the other groups.

Using these groups, we can now state which progressions are mutually compatible and which are not:

> **Assertion 5:** In a compatible sequence, all the progressions must come from the same group. Mixing progressions that are not in the same group will always cause one or more dancers to return to places they have already visited (which could, of course, be their home places), implying that those dancers will fail to get to all four places in four times through the dance.

In Group II, for example, the following three progressions are mutually compatible:

> Everyone moves one place to the left around the square.
> Everyone moves one place to the right around the square.
> Heads switch; side switch.

Inserting any other progression among these three will cause some dancers to go home early or revisit places they have already been before the dance is completed.[5]

This important assertion is a necessary condition for compatibility (it has to be true), but it is not sufficient, by itself, to guarantee that the dancers will actually get to all four places. That outcome depends on the *order* in which the progressions in a group are assembled.

> **Assertion 6:** Within each group, there are six ways to put the progressions into an order that will get each of the four dancers to each of the four place once and only once in four times through.

[5] This was the problem in the "pit fall" described earlier. The two progressions are from Groups II and III, respectively. Each will work satisfactorily with its group-mates, but they do not work with each other.

Each row in the following table gives a compatible sequence of progressions from *Group I*.[6]

Group I	1st	2nd	3rd	4th
Sequence 1	(1↔3)(2↔4)	(1↔4)(2↔3)	(1↔3)(2↔4)	(1↔4)(2↔3)
Sequence 2	(1↔4)(2↔3)	(1↔2)(3↔4)	(1↔4)(2↔3)	(1↔2)(3↔4)
Sequence 3	(1↔2)(3↔4)	(1↔3)(2↔4)	(1↔2)(3↔4)	(1↔3)(2↔4)
Sequence 4	(1↔3)(2↔4)	(1↔2)(3↔4)	(1↔3)(2↔4)	(1↔2)(3↔4)
Sequence 5	(1↔4)(2↔3)	(1↔3)(2↔4)	(1↔4)(2↔3)	(1↔3)(2↔4)
Sequence 6	(1↔2)(3↔4)	(1↔4)(2↔3)	(1↔2)(3↔4)	(1↔4)(2↔3)

Table 4

Sequence 1 says, for example, that progression (1↔3)(2↔4), followed by (1↔4)(2↔3), followed by (1↔3)(2↔4) again, followed by (1↔4)(2↔3) again will take four dancers to four places and home again in four moves. Sequence 6 is the one used in Table 3. *Notice that these sequences have an interesting pattern: In each one, the first two progressions are repeated as the second two progressions.* This alternating pattern will be useful in a moment.

Each row in the following table gives a compatible sequence of progressions from *Group II*.

Group II	1st	2nd	3rd	4th
Sequence 1	(1→2→3→4→)	(1→2→3→4→)	(1→2→3→4→)	(1→2→3→4→)
Sequence 2	(1→2→3→4→)	(1↔3)(2↔4)	(1→4→3→2→)	(1↔3)(2↔4)
Sequence 3	(1→4→3→2→)	(1→4→3→2→)	(1→4→3→2→)	(1→4→3→2→)
Sequence 4	(1→4→3→2→)	(1↔3)(2↔4)	(1→2→3→4→)	(1↔3)(2↔4)
Sequence 5	(1↔3)(2↔4)	(1→2→3→4→)	(1↔3)(2↔4)	(1→4→3→2→)
Sequence 6	(1↔3)(2↔4)	(1→4→3→2→)	(1↔3)(2↔4)	(1→2→3→4→)

Table 5

Sequences 1 and 3 are the *simple* compatible sequences for this group. Sequences 1 and 2 are the ones used in Tables 1 and 2, respectively.

If we did the same analysis for Groups III and IV, we could identify an additional 12 compatible sequences for a total of 24 in all. *These 24 sequences are the only ones that will satisfy the different places requirement.* Any progression order other than the six permitted within each group will cause one or more dancers to return to places they have already visited.

[6] The reasoning for determining which orders work correctly is not difficult. It is outlined in the Appendix under "Getting to All Four Places".

DIFFERENT PARTNER

We now need to identify *pairs* of compatible sequences–one for the men and one for the women–that satisfy the *different partners* requirement. The men and women must be routed through the dance so that no one meets any partner twice until the very end when original partners come together again in original places.

Definition: We will call pairs of compatible sequences that satisfy the different partners requirement ***sociable pairs***.

With so many compatible sequences to choose from among, one might expect it to be relatively easy to find sociable pairs. In fact, the choices are quite limited. The following three assertions (based on Theorem 2 in our first paper) explain why:

Assertion 7: To satisfy the different partners requirement, both members of a sociable pair *must* come from the same group.

In other words, choosing compatible sequences for the men and women from different groups will *always* cause at least some men and women to meet more than once.

Assertion 8: It can be shown that within a single *cycle* group (Groups II, III, or IV), there are *no* sociable pairs.

In other words, it is not possible to find a sociable matching between two compatible sequences taken from within the same cycle group. It does not matter if we use simple cycle sequences or mixed ones, they will never work.

These two assertions (no sociable matches *across* cycle groups or *within* cycle groups), bring us to a surprising result. Taken together, they contradict what most people (including the authors) might see as "common sense". They imply that

Assertion 9: *A DP/DP dance can never have a cycle progression in it.*

If even one cycle progression is included among the four used in a dance, the dance cannot be DP/DP!

What Are We Left With?

Having disqualified all of the cycle groups (Groups II, III, and IV), we are left with only Group I, composed entirely of exchange progressions, as a place where we might find DP/DP progressions. The following is true (Theorem 3):

Assertion 10: It can be shown that there are six pairs of compatible sequences from Group I that do in fact meet both the different places and different partners requirements.

These six sociable pairs are the *only* ones that will do the trick. They all use *mixes* of exchange progressions, which means that all DP/DP dances must have the alternating pattern noted earlier: The *progressions will be the same the first and third times through, and they will be the same the second and fourth times through.*

To identify the sociable pairs, look back at Table 4 (which was based in Group I). You may verify the following:

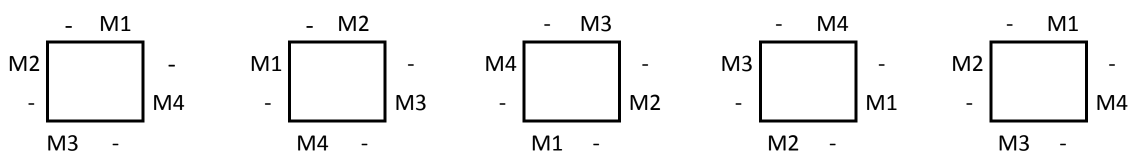
> **Assertion 11:** The top three compatible sequences in Table 4 are mutually sociable–any combination of two will work fine. Similarly, the bottom three compatible sequences are mutually sociable.

An Example

Suppose we try to devise a dance based on Table 4 using Sequence 4 for the women's travels and Sequence 6 men's travels. Sequence 6 is the one in Table 3 (reprinted below):

M's Progressions⇒	(1↔2)(3↔4)	(1↔4)(2↔3)	(1↔2)(3↔4)	(1↔4)(2↔3)	**#6**
Places	♬ **Home**	↘ ♬ ↗ **Arrangement 1**	↘ ♬ ↗ **Arrangement 2**	↘ ♬ ↗ **Arrangement 3**	**(Home ♬ Again) Arrangement 4**
1	M1	M2	M3	M4	M1
2	M2	M1	M4	M3	M2
3	M3	M4	M1	M2	M3
4	M4	M3	M2	M1	M4

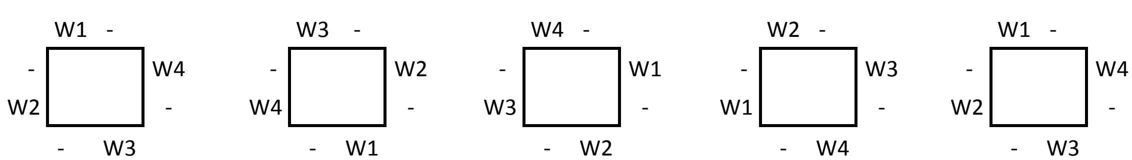
Table 6 **(=Table 3)**



Here is the comparable table for Sequence 4.

W's Progressions⇒	(1↔3)(2↔4)	(1↔2)(3↔4)	(1↔3)(2↔4)	(1↔2)(3↔4)	**#4**
Places	♬ **Home**	↘ ♬ ↗ **Arrangement 1**	↘ ♬ ↗ **Arrangement 2**	↘ ♬ ↗ **Arrangement 3**	**(Home ♬ Again) Arrangement 4**
1	W1	W3	W4	W2	W1
2	W2	W4	W3	W1	W2
3	W3	W1	W2	W4	W3
4	W4	W2	W1	W3	W4

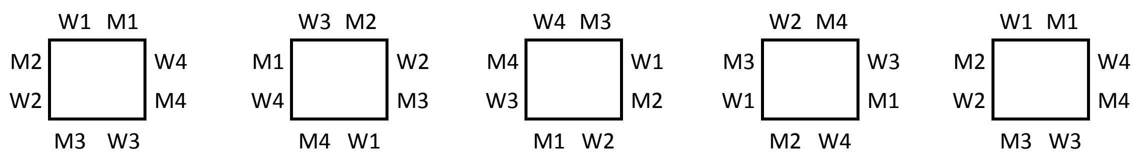
Table 7



Combining these two tables gives the follow result which does in fact satisfy different places and different partners requirements:

Places	♬ Home	♬ Arrangement 1	♬ Arrangement 2	♬ Arrangement 3	(Home ♬ Again) Arrangement 4
1	M1 W1	M2 W3	M3 W4	M4 W2	M1 W1
2	M2 W2	M1 W4	M4 W3	M3 W1	M2 W2
3	M3 W3	M4 W1	M1 W2	M2 W4	M3 W3
4	M4 W4	M3 W2	M2 W1	M1 W3	M4 W4

Table 8



This example shows one of the six possible solutions to the DP/DP problem. Two more solutions are described in the Appendix, and the dance in the next section, "The Invitation", uses still another solution. (And Example 9 in the first paper is one more example.)

The Appendix offers some extensions of the reasoning used here. We show that we could theoretically write a four-*trio* DP/DP dance for men, women *and children*; and we show that the cycle progressions that we set aside earlier are actually lurking in the background and playing an important role.

WRITING GOOD DP/DP DANCES

We have established now that it is *theoretically* possible to devise a four-couple dance that has the DP/DP property using sociable pairs from Group I. Once the required sequences are understood, it will almost always be possible for the choreographer to find *some* set of figures that will cause the progressions to occur. But finding those figures is only half the challenge for the choreographer who is striving to write dances that people will actually want to do. He or she will be concerned about whether the figures make sense to dancers. Will dancers be able to remember them? Will they be fun?

In all good dance writing, the challenge is to hide from dancers the complexities of what is really going on, so that everything seems logical and natural once it is understood. Nothing must seem contrived. This is difficult enough to achieve in writing ordinary dances, and it is more difficult to achieve in a DP/DP dance because, as we have seen, different figures must be done alternating times through the dance.

The ultimate judgement about whether DP/DP dances can be *good* dances will have to come from the dancers who do them. Nevertheless, it is worth noting that the first author has written several DP/DP dances that are in circulation (which is to say that someone other than the author has chosen to teach them). One of these dances, "The Invitation", is shown below. The first and second parts of the dance are the same every time through; the third and fourth parts alternate between two similar, but slightly different, sets of figures. One may verify that the underlying progressions are as follows:

Men's Progressions: (13)(24), (14)(23), (13)(24), (14)(23)
Women's Progressions: (12)(34), (13)(24), (12)(34), (13)(24)

(These are Sequences 1 and 3, respectively, in Table 4.)

There are other DP/DP dances (including some in longways formation) in the first author's new book, *A Group of Calculated Figures*, and on his web site.

http://home.earthlink.net/~gmrwebsite/

THE INVITATION
Square Formation
Historical English Style

This is a different-partners/different-places dance. It has *alternating B parts* which are similar, but not quite the same. With this property, it is possible for each person to dance from a different side of the set with a different partner each time through.

A	1-4	Taking hands-eight, circle halfway around.
	5-8	With Ptr, 2HT once around.
	9-12	Head couples pass through and courtesy turn.
	13-16	Side couples pass through and courtesy turn.

		1st and 3rd Times	2nd and 4th Times
B	1-2	Head couples lead out to the *right* and face side couples squarely.	Head couples lead out to the *left* and face side couples squarely.
	3-4	With opposite, *RHT* halfway.	With opposite, *LHT* halfway.
	5-8	Start circling hands four *left* and original head *men* lead your lines back out into a square (*men* are home, *wo-men* have changed places on the corners of the square).	Start circling hands four *right* and original head *women* lead your lines back out into a square (*women* are home, *men* have changed places on the corners of the square).
	9-10	Men *LHA* halfway and meet new Ptr (quick).	Women *RHA* halfway and meet new Ptr (quick).
	11-12	With new Ptr, set R&L.	With new Ptr, set R&L.
	13-16	With new Ptr, back-to-back.	With new Ptr, back-to-back.

M = Seq. 1 W = Seq. 3

MUSIC: “The Invitation” by Peter Barnes.

From *A Group of Calculated Figures* by Gary Roodman, 2012.

APPENDIX

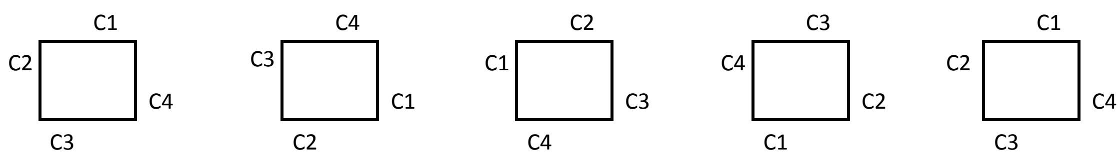
This appendix offers some additional observations about the example in main paper. The stuff here is not mathematically advanced, but it may require a little concentration to get through.

Family Dancing

Recall that Sequences 4, 5, and 6 in Table 4 are *mutually* sociable, meaning that any two of them will form a sociable pair. The example in the main body of this paper is based on using Sequences 4 and 6 as the sociable pair. Suppose we add a third set of dancers who follow Sequence 5. We will call them "Children" and denote them as C1, C2, C3, and C4. Their travels through the dance will be as follows:

C's Progressions⇒	(1↔4)(2↔3)	(1↔3)(2↔4)	(1↔4)(2↔3)	(1↔3)(2↔4)	**#5**
Places	**♬ Home**	**↘ ♬ ↗ Arrangement 1**	**↘ ♬ ↗ Arrangement 2**	**↘ ♬ ↗ Arrangement 3**	**(Home ♬ Again) Arrangement 4**
1	C1	C4	C2	C3	C1
2	C2	C3	C1	C4	C2
3	C3	C2	C4	C1	C3
4	C4	C1	C3	C2	C4

Table 9



You may verify that matching Table 9 with either Table 6 or Table 7 will give two additional DP/DP solutions. *And* there is one further possibility: Tables 6, 7, and 9 could theoretically be combined into one grand table of four DP/DP trios:

Grand Combined Dance Trios

Places	**♬ Home**	**♬ Arrangement 1**	**♬ Arrangement 2**	**♬ Arrangement 3**	**(Home ♬ Again) Arrangement 4**
1	M1 W1 C1	M2 W3 C4	M3 W4 C2	M4 W2 C3	M1 W1 C1
2	M2 W2 C2	M1 W4 C3	M4 W3 C1	M3 W1 C4	M2 W2 C2
3	M3 W3 C3	M4 W1 C2	M1 W2 C4	M2 W4 C1	M3 W3 C3
4	M4 W4 C4	M3 W2 C1	M2 W1 C3	M1 W3 C2	M4 W4 C4

Table 10

Note that no two triplets in this table are the same. Every dancer meets every other dancer exactly once.[7] Thus, if four nuclear families–one mom, one dad, one child–were doing

[7] Tables 6, 7, and 9, taken together, have a special name in mathematics. They are call *mutually orthogonal Latin squares*. Mathematicians, scientists, researchers, and others have found lots of uses for MOLS over the years. This is just one more place where MOLS have a useful interpretation. You may read more about MOLS in our first paper.

a four-trio dance with these progressions, every child would meet every mom and every dad; but, except at the beginning and end (when he is with his own parents), the moms and dads he meets each time will not be married couples.[8]

Dancers' Routes

This paper has approached the DP/DP problem by focusing on the *progressions* that are needed to create the desired results:

> "Everyone do this the first time, then everyone do something else the second time, ... "

This approach is the easiest way to formulate prescriptions for the choreographer who wants to write a DP/DP dance. But there is an alternative view of the problem that is also useful. It focuses on the *routes* that dancers travel:

> "1st dancer go to the four places in this order; 2nd dancer go the four places in a this (different) order, ..."

Look again at Tables 6, 7, and 9 and focus on how the individual dancers are moving through the four *places*:

Table 6 (Seq. 6) (1↔2)(3↔4) (1↔4)(2↔3)	M1: (1→2→3→4→) M2: (2→1→4→3→) M3: (3→4→1→2→) M4: (4→3→2→1→)	**M1 and M3** are traveling the cycle **(1→2→3→4→)**, but out of phase with each other. **M2 and M4** are traveling that same cycle *in the reverse direction*, again out of phase with each other.
Table 7 (Seq. 4) (1↔3)(2↔4) (1↔2)(3↔4)	W1: (1→3→4→2→) W2: (2→4→3→1→) W3: (3→1→ 2→4→) W4: (4→2→1→3→)	**W1 and W4** are traveling the cycle (**1→3→4→2→**), but out of phase with each other. **W2 and W3** are traveling that same cycle *in the reverse direction*, again out of phase with each other.
Table 9 (Seq. 5) (1↔4)(2↔3) (1↔3)(2↔4)	C1: (1→4→2→3→) C2: (2→3→1→4→) C3: (3→2→4→1→) C4: (4→1→3→2→)	**C1 and C2** are traveling the cycle **(1→4→2→3→)**, but out of phase with each other. **C3 and C4** are traveling that same cycle *in the reverse direction*, again out of phase with each other.

This view of the problem gives us an alternative way to describing how a DP/DP dance is constructed. *Pairs of cycles* (which are the inverses of each other) replace *pairs of progressions.* In each case, two dancers travel one route, and the other two travel the reverse route.

[8] We are working right now on writing a four-trio dance for moms, dads, and kids.

Getting To All Four Places

In the body of this paper, we asserted that

1. All the progressions in a compatible sequence must come from the same group, and

2. Within each group, there are six ways to put the progressions into an order that assures that each dancer gets to each place once and only once in four times through. Tables 4 and 5 enumerated the six possibilities for Groups I and II.

In this section, we show how those two sets of six sequences are generated. To explain the approach we need two new terms:

An ***arrangement*** specifies *where the four dancers are located* after some progressions have been done (as opposed to a *progression* which specifies a transition from one time through to the next).

Every progression and arrangement has an ***inverse*** that reverses its effect. For example, (1→2→3→4→) and (1→4→3→2→) simply "undo" each other; and (1↔2)(3↔4) is its own inverse.

As we put the progressions in order, we need to make sure that dancers (a) are kept away from home until the very last time through and (b) never go back to a place they have already visited. This can be done by following two simple rules: At any point in a dance,

(a) the current *arrangement* must not be followed immediately by its inverse (which would take everyone home), *except* the very last time through when we *want* everyone to go home, and

(b) no *progression* can be followed immediately by its inverse, since that would "undo" that progression and take dancers back to the places they occupied the time before last.

Here is a step-by-step plan:

1st Progression	Choose any progression in the selected group. (There will be three possibilities.)
2nd Progression	Choose any progression that is not the inverse of the 1st progression. (There will always be two possibilities here.)
3rd Progression	Choose the progression that is not the inverse of the 2nd progression *and* not the inverse of the current arrangement. (There will be only *one* possibility here.)
4th Progression	Choose the progression that *is* the inverse of the current arrangement. (Again, just one possibility.)

Notice that after the first two progressions are chosen, the other two are completely determined.