The DC Isolated 1:1 Guanella Transmission Line Transformer

by

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Introduction

The concept of transmission line transformers (TLTs) has been a distinct element of RF circuit design at least since 1944 when Guanella disclosed an impedance transformer of novel design which consisted of a pair of interconnected transmission lines [1, 2]. Subsequent to that disclosure, Guanella later showed that TLTs with DC isolation could be realized [3], as have others in more recent years [4, 5, 6].

Fundamental Concepts

The TLT operates by transmitting energy by way of the transverse (or TEM, meaning ***T****ransverse* ***E****lectro****M****agnetic* [7], also known as ***T****ransverse* ***E****lectric and* ***M****agnetic* [8]) transmission line môde, rather than on the more familiar coupling of flux as with a conventional transformer [9], and Fig. 1 illustrates this concept in generalised form, where the two lines represent the two conductors of a transmission line, regardless of whether it is made of parallel wires, twisted wires, coaxial cable, or any other means. Here, the currents in the two conductors are equal in magnitude and opposite in phase, while the voltages across the ends of the two conductors are equal in both magnitude and relative phase. In the TLT, the windings serve to eliminate, or at least substantially reduce common-môde currents from the input to the output [10].

For purposes of circuit analysis, the TEM transmission line of Fig. 1 can be described as a 2-port ABCD matrix, as shown in Fig. 2 [11], and mathematically (from Appendix A) as:

$$\begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \begin{bmatrix} Cosh\ \gamma l & Z_0\ Sinh\ \gamma l \\ \dfrac{Sinh\ \gamma l}{Z_0} & Cosh\ \gamma l \end{bmatrix} \times \begin{bmatrix} V_{out} \\ I_{out} \end{bmatrix} \qquad (1)$$

where l is the length of the transmission line and γ is the *complex propagation constant* :

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (2)$$

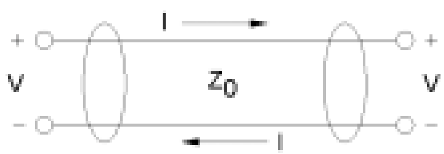

Figure 1 - Definition of Transmission Line in Transverse (TEM) Môde

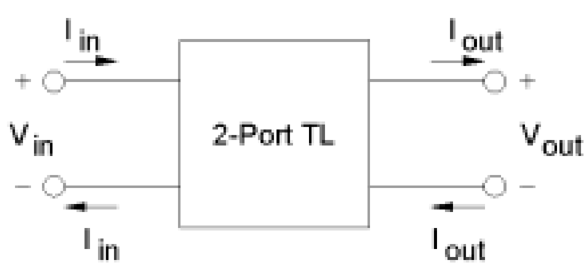

Figure 2 - 2-Port ABCD Depiction of TEM Transmission Line

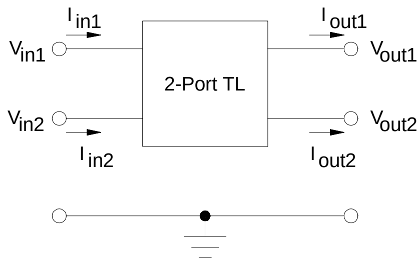

Figure 3 - 2-Port TEM Transmission Line as 4-Port ABCD Element

where α is called the *attenuation constant* and β is called the *phase constant*. The constants R, L, G, and C are R, L, G, and C are the total series resistance, series inductance, shunt conductance, and shunt capacitance per unit length of the transmission line, from which we can also derive the *characteristic impedance* Z_o:

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (3)$$

The 2-port transmission line of Fig. 2 may also be described as a 4-port ABCD matrix [12] where each terminal of the transmission line is paired with ground, as shown in Fig. 3, the mathematical description of which appears in Appendix B.

Typical 1:1 Transmission Line Transformer Applications

Fig. 4 illustrates a typical application of TEM transmission line, where a voltage generator having a source impedance Z_0 is coupled to a load impedance R_1 by way of a length of transmission line having a characteristic impedance of Z_0. With the same side of the transmission line grounded at both ends, the voltages across both ends are equal in magnitude (assuming that the line is lossless), although they will differ in absolute phase depending on the electrical length of the line. The input impedance and output voltage for Fig. 4 are:

$$Z_{in} = Z_0\left(\frac{R_1 \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + R_1 \sinh \gamma l}\right) \qquad (4)$$

$$V_1 = \frac{R_1 V_{in}}{R_1 \cosh \gamma l + Z_0 \sinh \gamma l} \qquad (5)$$

which are developed in Appendix A for a 2-port network and in Appendic C for a 4-port network. Note that equations (A4) through (A8) and (C7) through (C11) are the same, showing that the 2-port and 4-port approaches are identical.

Fig. 5 illustrates an application where a length of transmission line is used as a 1:1 phase inverting transformer. Here, the application shown in Fig. 4 has been modified by switching the connections on the load end, thereby causing the load to see the inverse of the input voltage. This application is used to realise inverting transformers for pulses having very high rise times [12]. The input impedance and output voltage for Fig. 5 are:

$$Z_{in} = Z_0\left(\frac{R_2 \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + R_2 \sinh \gamma l}\right) \qquad (6)$$

$$V_2 = \frac{-R_2 V_{in}}{R_2 \cosh \gamma l + Z_0 \sinh \gamma l} \qquad (7)$$

which are developed in Appendix D. It is interesting to note that the input impedance for

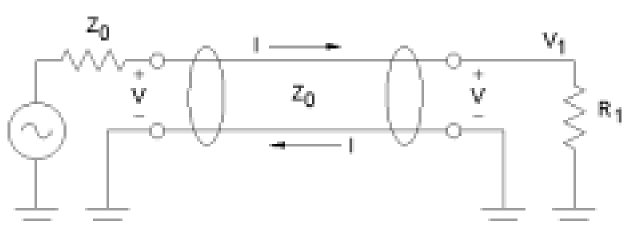

Figure 4 - Typical Application of TEM Transmission Line

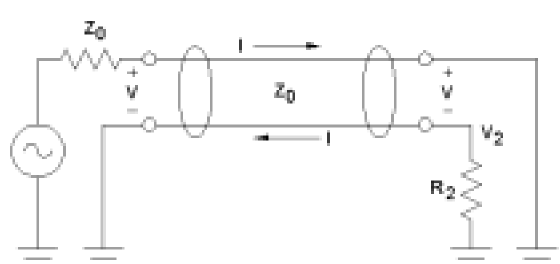

Figure 5 - Transmission Line Used as a 1:1 Phase Inverter

Fig. 5 is the same as that for Fig. 4, and that the output voltage for Fig. 5 is equal and opposite to that of Fig. 4.

This application can be verified by way of a very simple test, which consists of connecting a length of 50Ω coaxial cable to a signal generator and then connecting the free end to a 50Ω load resistor and ground as shown in Fig. 3. With the generator set to a frequency that is above the low frequency bandwidth limit for the length of cable, the voltage across the load resistance can be measured with an oscilloscope and observed to be equal in magnitude and opposite in phase to the input voltage.

Fig. 6 illustrates an application where a length of transmission line is used as a 1:1 choke balun, and is shown here with a balanced (or symmetrical) load. With the output currents being equal and opposite, the output voltages across the balanced load are also equal and opposite. The input impedance and output voltages for Fig. 6 are:

$$Z_{in} = Z_0 \left(\frac{(R_1+R_2)cosh\ \gamma l + Z_0 sinh\ \gamma l}{Z_0 cosh\ \gamma l + (R_1+R_2) sinh\ \gamma l} \right) \quad (8)$$

$$V_1 = \frac{R_1 V_{in}}{(R_1+R_2)cosh\ \gamma l + Z_0 sinh\ \gamma l} \quad (9)$$

$$V_2 = \frac{-R_2 V_{in}}{(R_1+R_2)cosh\ \gamma l + Z_0 sinh\ \gamma l} \quad (10)$$

the development of which appears in Appendix E.

Fig. 7 shows the choke balun of Fig. 6 in a more tyical application which is a very familar amongst radio amateurs where it is used extensively for matching unbalanced coaxial cable to balanced and floating loads, such as dipole antennas. Here, the voltage across the load is equal to that of the input voltage (less any losses), and is symmetrical with respect to ground. The input impedance and output voltages of Fig. 7 are:

$$Z_{in} = Z_0 \left(\frac{R_L cosh\ \gamma l + Z_0 sinh\ \gamma l}{Z_0 cosh\ \gamma l + R_L sinh\ \gamma l} \right) \quad (11)$$

$$V_1 = \frac{R_L}{2} \frac{V_{in}}{R_L cosh\ \gamma l + Z_0 sinh\ \gamma l} \quad (12)$$

$$V_2 = -\frac{R_L}{2} \frac{V_{in}}{R_L cosh\ \gamma l + Z_0 sinh\ \gamma l} \quad (13)$$

the development of which which appears in Appendix F, and which follows that of the mathematical development of Fig. 6, which appears in Appendix E, by realizing that with the two output currents being equal in magnitude and opposite in phase, a virtual ground exists at the centre of the load impedance. Therefore, in Appendix F the impedance $R_L/2$ is substituted for the load impedances R_1 and R_2 of Appendix E.

These applications can be verified by way of a very simple test, which consists of connecting a length of 50Ω coaxial cable to a signal generator and then connecting the free end to a pair of 24Ω resistors from each conductor to ground, as shown in Fig. 6. With the generator set to a frequency that is above the low frequency bandwidth limit for the length of cable, the voltage across the load

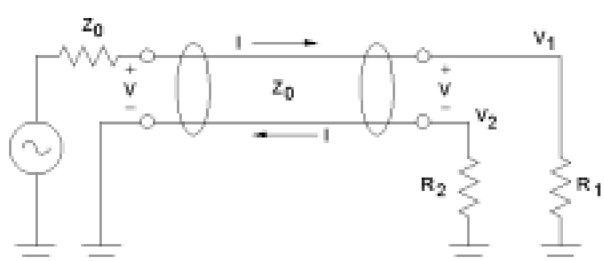

Figure 6 - Transmission Line Used as a 1:1 Choke Balun with a Balanced Load

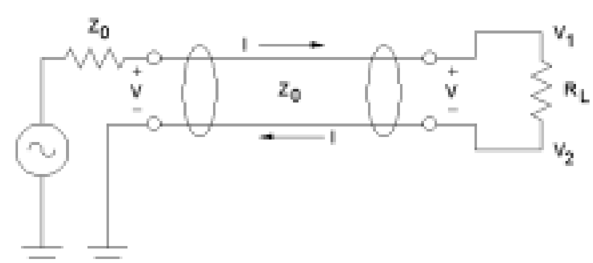

Figure 7 - Transmission Line Used as a 1:1 Choke Balun with a Floating Load

resistors can be measured with an oscilloscope and observed to be equal in magnitude and opposite in phase.

The choke balun of Fig. 7 can be tested using the setup for testing the choke balun of Fig. 6 by simply disconnectiong the common point between the two resistors from ground. This is a bit more difficult to test as any stray loading on either of the output terminals, even that from a scope probe, will upset the balance of the output voltages, although it is still functioning as TEM transmission line and therefore the voltages across both ends of the cable will remain equal.

In practice, choke baluns such as this are made appreciably shorter by placing a ferrite sleeve over the coaxial cable, which serves to suppress common-môde currents and which therefore makes the cable appear to be much longer by way of the approximation:

$$l' = l\sqrt{\mu_r} \tag{14}$$

where l' is the apparent length of the transmission line, l is the actual physical length, and μ_r is the relative permeability of the ferrite material.

At this point, we have come to realize an number of order-1 transmission line transformers which are easy to comprehend by way of the fact that the input voltage is applied differentially across the terminals on the input (left-hand) side and the load is connected

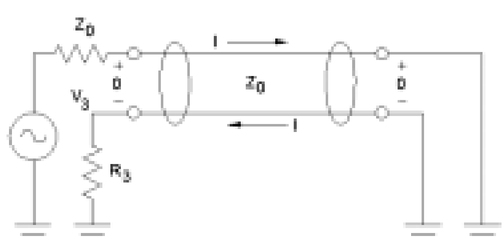

Figure 8 - TEM Transmission Line Used as a 1:1 Non-Inverting Transformer

differentially across the output (right-hand) side, and is connected in a way that causes the output voltage to be equal and opposite to the input voltage.

Now, Fig. 8 illustrates an application where a length of transmission line is used as a 1:1 non-inverting transformer. Here, the load impedance is attached to the input (left-hand) end of the lower conductor and on the right-hand end both conductors are connected to ground. The input impedance and output voltage for Fig. 8 are:

$$Z_{in} = \frac{V_{in}}{I_{in}} = Z_0 \frac{\sinh \gamma l}{\cosh \gamma l} = R_3 \tag{15}$$

$$V_3 = R_3 I_{in} = R_3 I_{out} \cosh \gamma l = = \frac{V_{in} R_3 \cosh \gamma l}{Z_0 \sinh \gamma l} = V_{in} \tag{16}$$

the development of which appears in Appendix G.

This application can be verified by way of a very simple test, which consists of connecting the centre conductor of a length of 50Ω coaxial cable to a signal generator and then connecting the outer conductor of the same end to a 50Ω load resistor, then connecting both conductors of the free end to ground as shown in Fig. 8. With the generator set to a frequency that is above the low frequency bandwidth limit for the length of cable, the voltage across the load resistance can be measured with an oscilloscope and observed to be equal in magnitude and phase to the input voltage.

The DC-Isolated 1:1 Guanella Transmission Line Transformer

We will now extend our understanding of 1:1 TLTs and consider the DC isolated 1:1 TLT shown in Fig. 9. Here, the load impedance is divided in half and applied to each end of the lower (or secondary) side of the transmission line, thereby creating a balanced (or symmetrical) load. Observe very carefully that the currents in the two sides of the transmission line remain equal and opposite at both ends, and that the voltages across the ends of the transmission line also remain equal, which is in total agreement with the theory of TEM transmission line. The input impedance and output voltages of Fig. 9 are:

$$Z_{in} = \frac{Z_0\left(R_2 + R_3\right)\cosh \gamma l + R_2 R_3 \sinh \gamma l}{Z_0 \cosh \gamma l - R_2 \sinh \gamma l} \quad (17)$$

$$V_2 = \frac{-R_2 V_{in} Z_0}{Z_0\left(R_2 + R_3\right)\cosh \gamma l + R_2 R_3 \sinh \gamma l} \quad (18)$$

$$V_3 = \frac{V_{in} R_3 \left(Z_0 \cosh \gamma l + R_2 \sinh \gamma l\right)}{Z_0\left(R_2 + R_3\right)\cosh \gamma l + R_2 R_3 \sinh \gamma l} \quad (19)$$

the development of which appears in Appendix H. At frequencies where the transmission line is short with respect to wavelength and the two load resistances are of equal value, the output voltages become of equal magnitude and opposite phase. The sum of these two voltages is equal to the input voltage, which is to be expected as the transmission line is functioning in TEM môde.

Fig. 10 shows the 1:1 Guanella isolation transformer of Fig. 9 with a floating load, where the secondary side is now is DC isolated from the primary side by simply removing the ground connection common to the two load resistors of Fig. 9, which is a more typical application. When the DC isolated 1:1 TLT balun of Fig. 9 is modified by replacing the two loads with a single floating load as shown in Fig. 10, the mathematical analysis becomes more demanding as now the input and output currents are forced to be equal as well as the voltages at each end of the transmission line, resulting in the input impedance and output voltages being:

$$Z_{in} = \frac{Z_0 R_L \cosh \gamma l + \left(n\,(1-n)\right) R_L^2 \sinh \gamma l}{\left(Z_0 \cosh \gamma l + n R_L \sinh \gamma l\right)} \quad (20)$$

$$V_2 = \frac{-V_{in}\, n R_L Z_0}{Z_0 R_L \cosh \gamma l + \left(n\,(1-n)\right) R_L^2 \sinh \gamma l} \quad (21)$$

$$V_3 = \frac{V_{in}(1-n) R_L Z_0}{Z_0 R_L \cosh \gamma l + \left(n\,(1-n)\right) R_L^2 \sinh \gamma l} \quad (22)$$

where

$$n = \frac{Z_0}{R_L} \tanh \frac{\gamma l}{2} \quad (23)$$

the development of which appears in Appendix J. The mathematical development of Fig. 10 was aided by observing that in Fig. 7 the output currents were equal in magnitude and opposite in phase, thereby creating a virtual ground at the centre of the load impedance for all frequencies.

A similar approach was used with Fig. 10 by observing that for frequencies where the length of the transmission line is small compared with wavelength, the output currents of Fig. 9 are equal and opposite. Relating this condition to Fig. 10, we set an arbitrary virtual ground between the ends of the load impedance. However, unlike the analisis of Fig. 7 (Appendix F), this condition will not

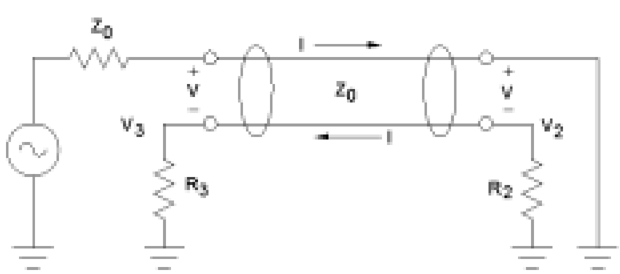

Figure 9 - Transmission Line Used as a 1:1 Guanella Isolation Transformer with a Balanced Load

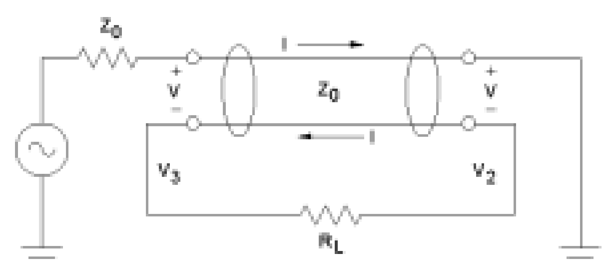

Figure 10 - Transmission Line Used as a 1:1 Guanella Isolation Transformer with a Floating Load

persist for all frequencies, and at low frequencies the output voltages for Fig. 10 are much the same as for the noninverting TLT transformer of Fig. 8 while at high frequencies the output voltages become much the same as for the inverting TLT transformer of Fig. 5.

By inspecting the equations for the currents and voltages for Fig. 9 in Appendix H, we observe that we can adjust the virtual ground of the load impedance of Fig. 10 so that the two input currents become equal in magnitude and opposite in phase while at the same time keeping the voltages across the ends of the transmission line equal. This is eased by further observing that the denominators of the equations (H10) and (H12) are identical, and it is therefore only necessary that an equality of the numerators be performed.

For this purpose, an arbitrary scalar **n** is introduced that serves to divide the load impedance into two separate real impedances, with their common point being a virtual ground. This equality is begun with equation (J7) and proceeds to equation (J11), where it is shown that for frequencies where the length of the line becomes insignificant and the load impedance is equal to Z_0, the value of the scalar is zero, which results in the left-hand output voltage being equal to the input voltage and the right-hand output voltage being zero.

We should also observe that in both Fig. 9 and Fig. 10 the input voltage is applied differentially across the input (again left-hand) side and it is equal in magnitude to the the output (again right-hand) voltage, which is in perfect agreement with the basic concept of TEM transmission line that is more readily understood.

The application of Fig. 9 can also be verified by way of a very simple test, which consists of connecting the centre conductor of a length of 50Ω coaxial cable to a signal generator and then connecting the centre conductor of the free end to ground as shown in Fig. 9. Now, both ends of the outer conductor (or shield) are connected to a pair of 24Ω resistors and the opposite ends of the resistors are connected to ground. With the generator again set to a frequency that is above the low frequency bandwidth limit for the length of cable, the voltage across the two load resistors can be measured with an oscilloscope and observed to each be half the magnitude of the input voltage and of opposite phase, and that their sum is equal to the input voltage.

As with the choke balun of Fig. 7. the DC isolated 1:1 Guanella transformer of Fig. 10 can be tested using the setup for testing the choke balun of Fig. 9 by simply disconnectiong the common point between the two resistors from ground. Also as with the choke balun of Fig. 7, the DC isolated 1:1 Guanella transformer of Fig. 10 is a bit more difficult to test any stray loading on either of the output terminals, even that from a scope probe, will upset the balance of the output voltages, although it is still functioning as TEM transmission line as the voltage across the load resistor is equal to the input voltage.

Closing Remarks

The design and application of DC isolated transmission line transformers (TLTs) requires a thorough understanding of the fundamentals of transmission line theory, 4-port network analysis, and the design of TLTs. The comprehension of the DC isolated TLT requires nothing more than a logical extension of these basic concepts, and once that is done they are readily and easily understood even by those having entry level experience in the profession of RF circuit design.

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Appendix A
TEM Transmission Line as 2-Port ABCD Network

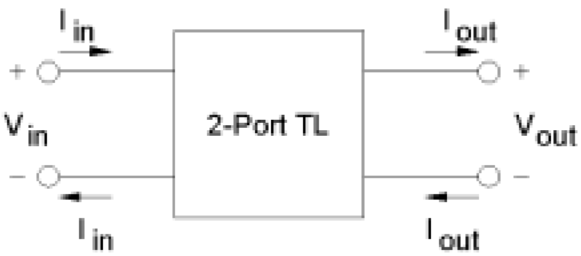

Figure 11 - 2-Port Depiction of TEM Transmission Line

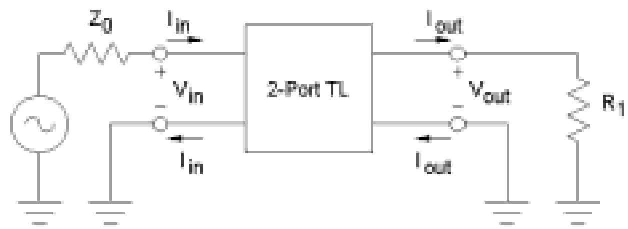

Figure 12 - 2-Port TEM Transmission Line with Source and Load

$$\begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \begin{bmatrix} Cosh\ \gamma\ell & Z_0\ Sinh\ \gamma\ell \\ \dfrac{Sinh\ \gamma\ell}{Z_0} & Cosh\ \gamma\ell \end{bmatrix} \times \begin{bmatrix} V_{out} \\ I_{out} \end{bmatrix} \quad (A1)$$

$$V_{in} = V_{out}\, sinh\ \gamma\ell + I_{out}\, cosh\ \gamma\ell \quad (A2)$$

$$I_{in} = \frac{V_{out}}{Z_0}\ sinh\ \gamma\ell + I_{out}\, cosh\ \gamma\ell \quad (A3)$$

$$V_{out} = R_1\, I_{out} \quad (A4)$$

$$V_{in} = Z_0\, I_{out}\, sinh\ \gamma\ell + R_1\, I_{out}\, cosh\ \gamma\ell = \\ = I_{out}\left(Z_0\, sinh\ \gamma\ell + R_1 cosh\ \gamma\ell\right) \quad (A5)$$

$$I_{in} = \frac{R_1}{Z_0}\ I_{out}\, sinh\ \gamma\ell + I_{out}\, cosh\ \gamma\ell = \\ = I_{out}\left(\frac{Z_0\, cosh\ \gamma\ell + R_1 sinh\ \gamma\ell}{Z_0}\right) \quad (A6)$$

$$Z_{in} = Z_0\left(\frac{R_1 cosh\ \gamma\ell + Z_0\, sinh\ \gamma\ell}{Z_0\, cosh\ \gamma\ell + R_1 sinh\ \gamma\ell}\right) \quad (A7)$$

$$I_{out} = \frac{V_{in}}{R_1 cosh\ \gamma\ell + Z_0\, sinh\ \gamma\ell} \quad (A8)$$

$$V_{out} = \frac{R_1\, V_{in}}{R_1 cosh\ \gamma\ell + Z_0\, sinh\ \gamma\ell} \quad (A9)$$

Appendix B
TEM Transmission Line as 4-Port ABCD Network

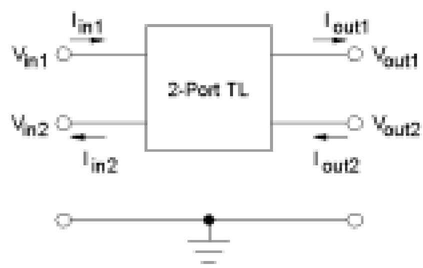

Figure 13 - 2-Port TEM Transmission Line as 4-Port ABCD Element

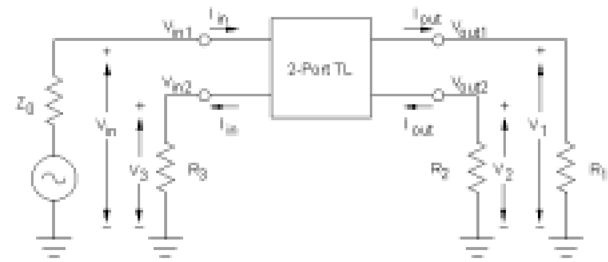

Figure 14 - 4-Port TEM Transmission Line with Source and Loads

$$V_{in} = V_1 \cosh \gamma l + Z_0 I_{out} \sinh \gamma l = R_1 I_{out} \cosh \gamma l + Z_0 I_{out} \sinh \gamma l = I_{out} \left(Z_0 \sinh \gamma l + R_1 \cosh \gamma l\right) \quad \text{(B11)}$$

$$V_3 = V_2 \cosh \gamma l + Z_0 I_{out} \sinh \gamma l = -R_2 I_{out} \cosh \gamma l + Z_0 I_{out} \sinh \gamma l = I_{out} \left(Z_0 \sinh \gamma l - R_2 \cosh \gamma l\right) = R_3 I_{in} \quad \text{(B12)}$$

$$I_{in} = \frac{\left(V_1 - V_2\right)}{Z_0} \sinh \gamma l + I_{out} \cosh \gamma l = I_{out} \frac{\left(R_1 - R_2\right)}{Z_0} \sinh \gamma l + I_{out} \cosh \gamma l = I_{out} \left(\frac{Z_0 \cosh \gamma l + \left(R_1 + R_2\right) \sinh \gamma l}{Z_0}\right) \quad \text{(B13)}$$

Appendix C
4-Port TEM Transmission Line Network as 2-Port TEM Transmission Line

$$R_2 = 0 \quad \text{(C1)}$$

$$R_3 = 0 \quad \text{(C2)}$$

$$V_2 = 0 \quad \text{(C3)}$$

$$V_3 = 0 \quad \text{(C4)}$$

$$V_{in} = I_{out}\left(Z_0 \sinh \gamma l + R_1 \cosh \gamma l\right) \quad \text{(C5)}$$

$$I_{in} = I_{out}\left(\frac{Z_0 \cosh \gamma l + R_1 \sinh \gamma l}{Z_0}\right) \quad \text{(C7)}$$

$$Z_{in} = Z_0 \left(\frac{R_1 \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + R_1 \sinh \gamma l}\right) \quad \text{(C8)}$$

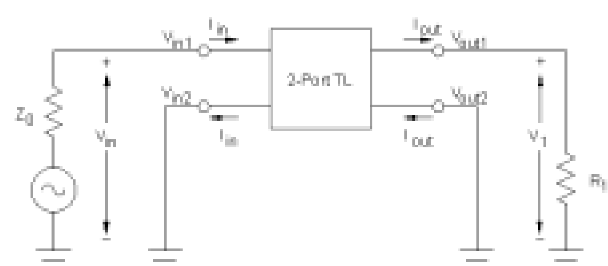

Figure 15 - 4-Port TEM Transmission Line as 2-Port TEM Transmission Line

$$I_{out} = \frac{V_{in}}{R_1 \cosh \gamma l + Z_0 \sinh \gamma l} \quad \text{(C9)}$$

$$V_1 = \frac{R_1 V_{in}}{R_1 \cosh \gamma l + Z_0 \sinh \gamma l} \quad \text{(C10)}$$

Appendix D
4-Port TEM Transmission Line Network as 1:1 TLT Inverting Transformer

$$R_1 = 0 \qquad \text{(D1)}$$

$$R_3 = 0 \qquad \text{(D2)}$$

$$V_1 = 0 \qquad \text{(D3)}$$

$$V_3 = 0 \qquad \text{(D4)}$$

$$V_{in} = I_{out}\left(R_2 \ \mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l\right) \qquad \text{(D5)}$$

$$I_{in} = I_{out}\left(\frac{Z_0\,\mathit{cosh}\ \gamma l + R_2\,\mathit{sinh}\ \gamma l}{Z_0}\right) \qquad \text{(D7)}$$

$$I_{out} = \frac{V_{in}}{R_2 \ \mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l} \qquad \text{(D8)}$$

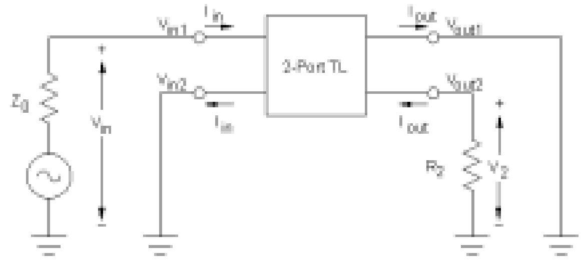

Figure 16 - 4-Port TEM Transmission Line as 1:1 TLT Inverting Transformer

$$Z_{in} = Z_0\left(\frac{R_2\,\mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l}{Z_0\,\mathit{cosh}\ \gamma l + R_2\,\mathit{sinh}\ \gamma l}\right) \qquad \text{(D9)}$$

$$V_2 = \frac{-R_2\,V_{in}}{R_2\,\mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l} \qquad \text{(D10)}$$

Appendix E
4-Port TEM Transmission Line Network as 1:1 TLT Choke Balun

$$R_3 = 0 \qquad \text{(E1)}$$

$$V_3 = 0 \qquad \text{(E2)}$$

$$V_{in} = I_{out}\left(\left(R_1+R_2\right)\mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l\right) \qquad \text{(E3)}$$

$$I_{in} = I_{out}\left(\frac{Z_0\,\mathit{cosh}\ \gamma l + \left(R_1+R_2\right)\mathit{sinh}\ \gamma l}{Z_0}\right) \qquad \text{(E4)}$$

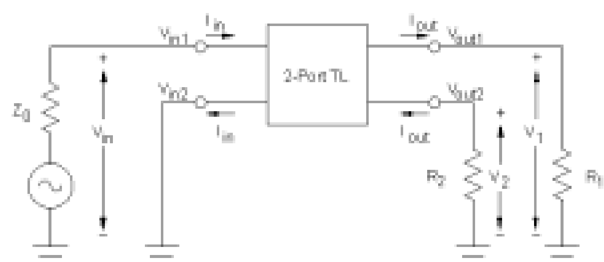

Figure 17 - TEM Transmission Line as 4-Port Choke Balun Network

$$Z_{in} = Z_0\left(\frac{\left(R_1+R_2\right)\mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l}{Z_0\,\mathit{cosh}\ \gamma l + \left(R_1+R_2\right)\mathit{sinh}\ \gamma l}\right) \qquad \text{(E6)}$$

$$I_{out} = \frac{V_{in}}{\left(R_1+R_2\right)\mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l} \qquad \text{(E7)}$$

$$V_1 = \frac{R_1\,V_{in}}{\left(R_1+R_2\right)\mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l} \qquad \text{(E8)}$$

$$V_2 = \frac{-R_2\,V_{in}}{\left(R_1+R_2\right)\mathit{cosh}\ \gamma l + Z_0\,\mathit{sinh}\ \gamma l} \qquad \text{(E9)}$$

Appendix F
4-Port TEM Transmission Line Network as 1:1 TLT Choke Balun with Floating Load

$$R_1 = R_2 = \frac{R_L}{2} \tag{F1}$$

$$R_3 = 0 \tag{F2}$$

$$V_3 = 0 \tag{F3}$$

$$V_{in} = I_{out}\left(R_L \cosh \gamma l + Z_0 \sinh \gamma l\right) \tag{F4}$$

$$I_{in} = I_{out}\left(\frac{Z_0 \cosh \gamma l + R_L \sinh \gamma l}{Z_0}\right) \tag{F5}$$

$$Z_{in} = Z_0\left(\frac{R_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + R_L \sinh \gamma l}\right) \tag{F6}$$

$$I_{out} = \frac{V_{in}}{R_L \cosh \gamma l + Z_0 \sinh \gamma l} \tag{F7}$$

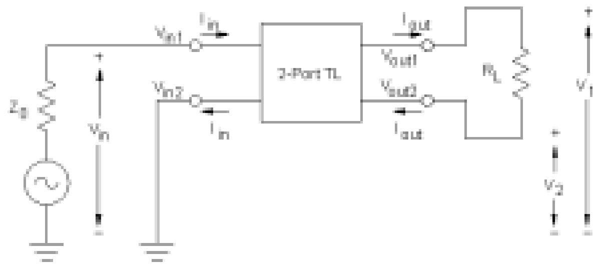

Figure 18 - TEM Transmission Line as 4-Port Choke Balun Network with Floating Load

$$V_1 = \frac{R_L}{2}\,\frac{V_{in}}{R_L \cosh \gamma l + Z_0 \sinh \gamma l} \tag{F8}$$

$$V_2 = -\frac{R_L}{2}\,\frac{V_{in}}{R_L \cosh \gamma l + Z_0 \sinh \gamma l} \tag{F9}$$

Appendix G
4-Port TEM Transmission Line Network as 1:1 TLT Non-Inverting Transformer

$$R_1 = 0 \tag{G1}$$

$$R_2 = 0 \tag{G2}$$

$$V_1 = 0 \tag{G3}$$

$$V_2 = 0 \tag{G4}$$

$$V_{in} = I_{out}\, Z_0 \sinh \gamma l \tag{G5}$$

$$I_{out} = \frac{V_{in}}{Z_0 \sinh \gamma l} = \frac{R_3\, I_{in}}{Z_0 \sinh \gamma l} \tag{G8}$$

$$I_{in} = I_{out} \cosh \gamma l = \frac{V_{in} \cosh \gamma l}{Z_0 \sinh \gamma l} \tag{G9}$$

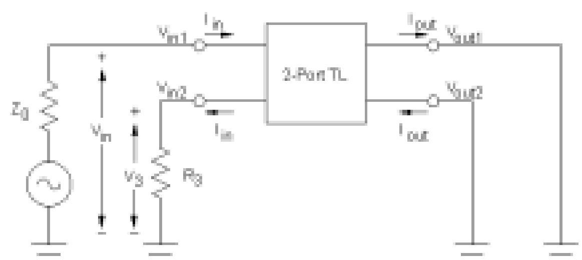

Figure 19 - 4-Port TEM Transmission Line as 1:1 TLT Non-Inverting Transformer

$$Z_{in} = \frac{V_{in}}{I_{in}} = Z_0 \frac{\sinh \gamma l}{\cosh \gamma l} = R_3 \tag{G10}$$

$$V_3 = R_3\, I_{in} = R_3\, I_{out} \cosh \gamma l = \frac{V_{in}\, R_3 \cosh \gamma l}{Z_0 \sinh \gamma l} = V_{in} \tag{G11}$$

Appendix H
4-Port TEM Transmission Line Network as
1:1 TLT Current Balun

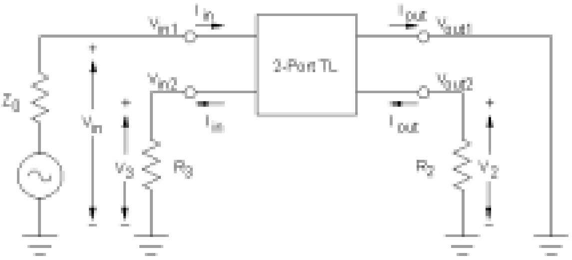

Figure 20 - TEM Transmission Line as 4-Port Current Balun Network

$$R_1 = 0 \qquad \text{(H1)}$$

$$V_1 = 0 \qquad \text{(H2)}$$

$$V_{in} = I_{out} Z_0 \, sinh \; \gamma l \qquad \text{(H3)}$$

$$R_3 \, I_{in} = I_{out} \left(Z_0 \, sinh \; \gamma l - R_2 \, cosh \; \gamma l \right) \qquad \text{(H4)}$$

$$R_3 \, I_{in} = V_{in} - I_{out} R_2 \, cosh \; \gamma l \qquad \text{(H5)}$$

$$I_{in} = I_{out} \left(\frac{R_2 \; sinh \; \gamma l + Z_0 \; cosh \; \gamma l}{Z_0} \right) = I_{out} \left(\frac{Z_0 \; sinh \; \gamma l - R_2 \; cosh \; \gamma l}{R_3} \right) \qquad \text{(H6)}$$

$$I_{out} = \frac{I_{in} Z_0}{R_2 \; sinh \; \gamma l + Z_0 \; cosh \; \gamma l} = \frac{I_{in} R_3}{Z_0 \; sinh \; \gamma l - R_2 \; cosh \; \gamma l} \qquad \text{(H7)}$$

$$V_{in} = R_3 \, I_{in} + I_{out} R_2 \, cosh \; \gamma l = R_3 \, I_{in} + \frac{I_{out} Z_0 R_2 \, cosh \; \gamma l}{R_2 \, sinh \; \gamma l + Z_0 \, cosh \; \gamma l} = \frac{I_{out} Z_0 R_2 \, cosh \; \gamma l + R_2 \, sinh \; \gamma l}{Z_0 (R_2 + R_3) cosh \; \gamma l + R_2 R_3 \, sinh \; \gamma l} \qquad \text{(H8)}$$

$$Z_{in} = \frac{Z_0 (R_2 + R_3) cosh \; \gamma l + R_2 R_3 \, sinh \; \gamma l}{Z_0 \, cosh \; \gamma l - R_2 \, sinh \; \gamma l} \qquad \text{(H9)}$$

$$I_{in} = \frac{V_{in} \left(Z_0 \, cosh \; \gamma l + R_2 \, sinh \; \gamma l \right)}{Z_0 (R_2 + R_3) cosh \; \gamma l + R_2 R_3 \, sinh \; \gamma l} \qquad \text{(H10)}$$

$$V_3 = \frac{V_{in} R_3 \left(Z_0 \, cosh \; \gamma l + R_2 \, sinh \; \gamma l \right)}{Z_0 (R_2 + R_3) cosh \; \gamma l + R_2 R_3 \, sinh \; \gamma l} \qquad \text{(H11)}$$

$$I_{out} = \frac{V_{in} Z_0}{Z_0 (R_2 + R_3) cosh \; \gamma l + R_2 R_3 \, sinh \; \gamma l} \qquad \text{(H12)}$$

$$V_2 = \frac{-R_2 V_{in} Z_0}{Z_0 (R_2 + R_3) cosh \; \gamma l + R_2 R_3 \, sinh \; \gamma l} \qquad \text{(H13)}$$

Appendix J
4-Port TEM Transmission Line Network as 1:1 TLT Current Balun with Floating Load

$$I_{in} = I_{out} = I \quad \text{(J1)}$$

$$R_1 = 0 \quad \text{(J2)}$$

$$V_1 = 0 \quad \text{(J3)}$$

$$R_2 = R_L \quad \text{(J4)}$$

$$R_3 = (1-n)\, R_L \quad \text{(J5)}$$

$$I_{out} = \frac{I_{in}\, Z_0}{R_2\; sinh\; \gamma l + Z_0\; cosh\; \gamma l} \quad \text{(J6)}$$

$$I\, Z_0 = I \left(n\, R_L\; sinh\; \gamma l + Z_0\; cosh\; \gamma l\right) \quad \text{(J7)}$$

$$n\, R_L\; sinh\; \gamma l = Z_0 \left(1 - cosh\; \gamma l\right) \quad \text{(J8)}$$

$$n = \frac{Z_0 \left(1 - cosh\; \gamma l\right)}{R_L\; sinh\; \gamma l} \geq 0 \quad \text{(J9)}$$

$$sinh\; \gamma l = \pm \sqrt{cosh^2\; \gamma l - 1} \quad \text{(J10)}$$

$$n = \frac{Z_0}{R_L} \left| \frac{cosh\; \gamma l - 1}{sinh\; \gamma l} \right| = \frac{Z_0}{R_L} \left(\frac{cosh\; \gamma l - 1}{\sqrt{cosh^2\; \gamma l - 1}} \right) =$$

$$= \frac{Z_0}{R_L} \left(\frac{\sqrt{(cosh\; \gamma l - 1)(cosh\; \gamma l - 1)}}{\sqrt{(cosh\; \gamma l - 1)(cosh\; \gamma l + 1)}} \right) =$$

$$= \frac{Z_0}{R_L} \left(\sqrt{\frac{cosh\; \gamma l - 1}{cosh\; \gamma l + 1}} \right) = \frac{Z_0}{R_L}\; tanh\; \frac{\gamma l}{2} \quad \text{(J11)}$$

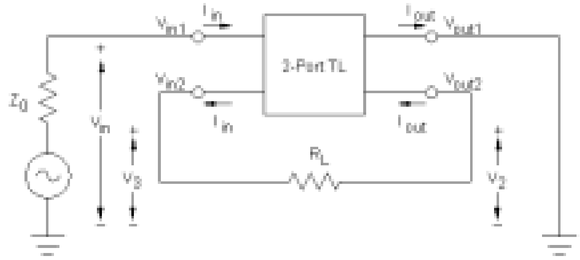

Figure 21 - TEM Transmission Line as 4-Port Current Balun Network with Floating Load

$$Z_{in} = \frac{Z_0\, R_L\, cosh\; \gamma l + \left(\, n\, (1-n)\,\right) R_L{}^2\, sinh\; \gamma l}{\left(Z_0\, cosh\; \gamma l + n\, R_L\, sinh\; \gamma l\right)} \quad \text{(J12)}$$

$$I_{in} = \frac{V_{in} \left(Z_0\, cosh\; \gamma l + n\, R_L\, sinh\; \gamma l\right)}{Z_0\, R_L\, cosh\; \gamma l + \left(\, n\, (1-n)\,\right) R_L{}^2\, sinh\; \gamma l} =$$

$$= \frac{V_{in}\, Z_0}{Z_0\, R_L\, cosh\; \gamma l + \left(\, n\, (1-n)\,\right) R_L{}^2\, sinh\; \gamma l} \quad \text{(J13)}$$

$$V_3 = \frac{V_{in} (1-n)\, R_L \left(Z_0\, cosh\; \gamma l + n\, R_L\, sinh\; \gamma l\right)}{Z_0\, R_L\, cosh\; \gamma l + \left(n\, (1-n)\right) R_L{}^2\, sinh\; \gamma l} =$$

$$= \frac{V_{in} (1-n)\, R_L\, Z_0}{Z_0\, R_L\, cosh\; \gamma l + \left(n\, (1-n)\right) R_L{}^2\, sinh\; \gamma l} \quad \text{(J14)}$$

$$I_{out} = \frac{V_{in}\, Z_0}{Z_0\, R_L\, cosh\; \gamma l + \left(n\, (1-n)\right) R_L{}^2\, sinh\; \gamma l} \quad \text{(J15)}$$

$$V_2 = \frac{-\, V_{in}\, n\, R_L\, Z_0}{Z_0\, R_L\, cosh\; \gamma l + \left(n\, (1-n)\right) R_L{}^2\, sinh\; \gamma l} \quad \text{(J16)}$$