Wideband Transformer Models: Measurement and Calculation of Reactive Elements

by

Chris Trask / N7ZWY
Sonoran Radio Research
P.O. Box 25240
Tempe, AZ 85285-5240

Senior Member IEEE

Email: christrask@earthlink.net

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Introduction

Wideband transformers are common elements in analogue circuit design ranging from audio frequencies to the UHF spectrum and very often beyond. Although the construction practices vary widely through this wide frequency spectrum, certain aspects of modeling are common throughout. The purpose of this paper is to discuss these common model elements in the broadest possible generalities, and from there discuss methods by which these model elements may be obtained by way of evaluations done at the transformer's input and output ports and a subsequent series of simple calculations.

Wideband Transformer Models

Transformers appear in schematics as a simple illustration of two or more windings on a common core of magnetic material, shown in the simplest form in Fig. 1. Here, a 2-winding transformer having a windings ratio of n:1 consists of a primary winding and a secondary winding, having primary and secondary inductances of L_P and L_S, respectively, as well as a mutual inductance M, which is determined by way of (1, 2, 3):

$$M = k\sqrt{L_P\, L_S} \tag{1}$$

where the *k* is known as the coupling coefficient. Neither the mutual inductance M nor the coupling coefficient *k* can be measured directly, but are instead determined by way of further evaluations of the transformer.

As shown in Fig. 2, each of the windings of the transformer of Fig. 1 consists of a winding inductance and a leakage inductance. For the primary winding, these individual inductances are related by way of:

$$L_P = L_{l1} + L_1 \tag{2}$$

where L_1 is the primary winding inductance and L_{l1} is the primary leakage inductance. Similarly, for the secondary winding the individual inductances are related by way of:

$$L_S = L_{l2} + L_2 \tag{3}$$

where L_2 is the primary winding inductance and L_{l2} is the primary leakage inductance. The mutual inductance M is related to the primary and secondary winding inductances by way of:

$$M = \sqrt{L_1\, L_2} \tag{4}$$

From Eq. 1 and Eq. 4, the coupling coefficient *k* can be determined by way of:

$$k = \sqrt{\frac{L_1\ L_2}{L_P\ L_S}} \tag{5}$$

In a further development of the wideband transformer model, parasitic capacitances are added, as shown in the complete model of the lossless wideband transformer of Fig. 3. Here,

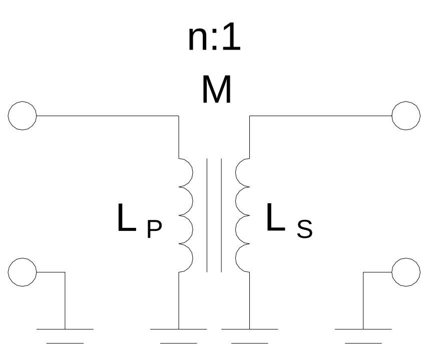

Fig. 1 - Circuit of 2-Winding Transformer

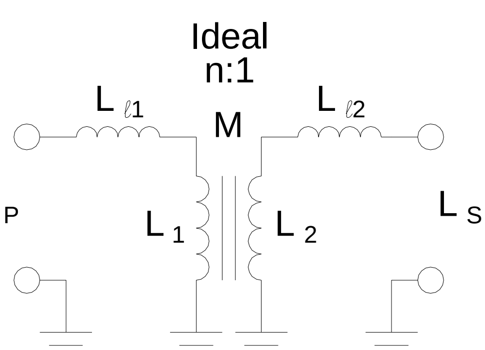

Fig. 2 - Circuit of 2-Winding Transformer Showing Winding and Leakage Inductances

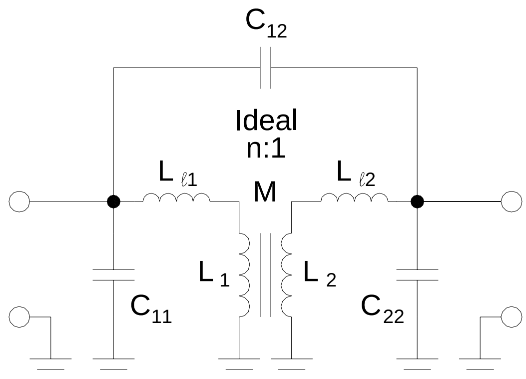

Fig. 3 - Complete Model of Lossless Wideband Transformer

C_{11} is the primary intrawinding capacitance, C_{22} is the secondary intrawinding capacitance, and C_{12} is the interwinding capacitance. The interwinding capacitance C_{12} serves to provide a transmission zero which is the primary limiting factor in the high frequency cutoff of the wideband transformer.

In circuit modeling, it is common practice to arrange the wideband transformer model so that it is referenced to one side or the other, as shown in Fig. 4 where the model of Fig. 3 has be referred to the primary side. Here, the model capacitances are (1, 4):

$$C'_{11} = C_{11} + C_{12}\left(1 - \frac{1}{n}\right) \tag{6}$$

$$C'_{12} = \frac{C_{12}}{n} \tag{7}$$

$$C'_{22} = \frac{C_{22}}{n^2} + C_{12}\left(\frac{1}{n} - 1\right) \tag{8}$$

Additional relationships between the wideband transformer model inductances are (8):

$$L_P = L_{l1} + \mathrm{n}\,M \tag{9}$$

$$M = \frac{L_P - L_{l1}}{n} \tag{10}$$

$$L_S = L_{l2} + \frac{M}{n} \tag{11}$$

$$M = n\left(L_S - L_{l2}\right) \tag{12}$$

More sophisticated models of wideband transformers are commonly found, such as that of Fig. 5 which includes the primary and secondary winding loss resistances R_P and R_S, respectively, as well as the core loss resistance R_C (1, 5). The winding loss resistances R_P and R_S are a result of the losses due to bulk resistance as well as skin effect of the conductors, the latter of which is frequency dependent (6). The core loss resistance R_C is a result of hysteresis and eddy current losses in the core material, which are both frequency and signal level dependent (7), as well as the magnetic skin effect of the core material, which is primarily frequency dependent.

The topic of losses in transformers is a

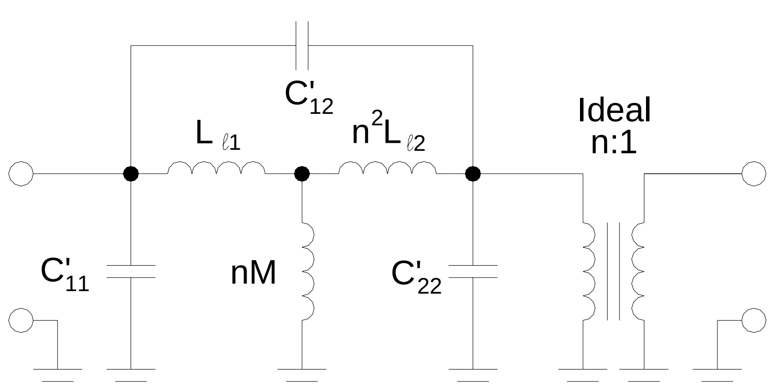

Fig. 4 - Complete Model of Lossless Wideband Transformer Referred to the Primary Side

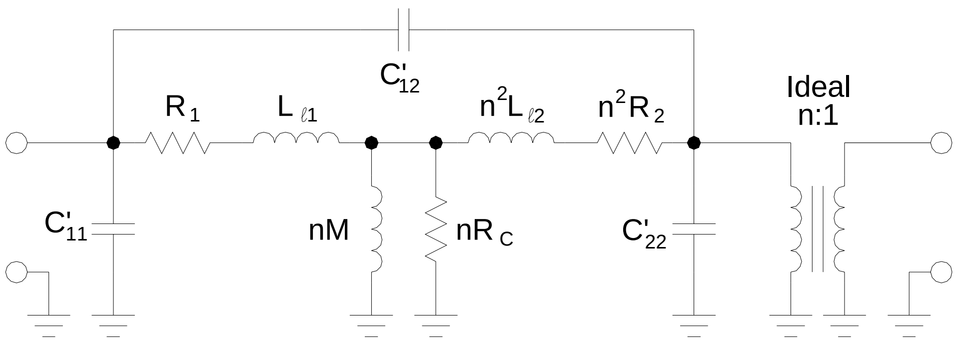

Fig. 5 - Complete Wideband Transformer Model

very advanced subject that is rarely dealt with except in cases where such losses are significant factors, such as in switch-mode power supplies and electrical power distribution. For most, but not all aspects of analogue and RF circuit design, these losses are relatively insignificant (6) and are rarely taken into consideration and are beyond the scope of this discussion.

Determining the Model Inductances

Determining the value of the inductances of the wideband transformer model of Fig. 4 begins by choosing a suitable midband test frequency where the effects of the capacitances are rendered as being insignificant. Referring then to the illustration of Fig. 4, the primary inductance L_P is measured across the primary terminals with the secondary terminals being open. An additional primary inductance L'_P is then measured with the secondary terminals shorted, which results in the equivalent circuit shown in Fig. 6.

Referring yet again to the illustration of Fig. 4, the secondary inductance L_S is measured across the secondary terminals with the primary terminals being open. An additional secondary inductance L'_S is then measured with the primary terminals shorted, which results in the equivalent circuit shown in Fig. 7.

From this series of measurements, the inductances of the wideband transformer model may be determined. Referring first to Fig. 7, we begin by recognizing that the value of L'_S can be determined by way of:

$$L'_S = L_{l2} + \frac{M\ L_{l1}}{\mathrm{n}^2 M + \mathrm{n}\ L_{l1}} \tag{13}$$

Substituting Eq. 12 for M in Eq. 13, we obtain:

$$\begin{aligned} L'_S &= L_{l2} + \frac{\left(\dfrac{L_P - L_{l1}}{\mathrm{n}}\right) L_{l1}}{\mathrm{n}\left(L_P - L_{l1}\right) + \mathrm{n}\ L_{l1}} = \\ &= L_{l2} + \frac{\left(L_P - L_{l1}\right) L_{l1}}{\mathrm{n}^2\ L_{l1}} = \\ &= L_{l2} + \frac{L_{l1}}{\mathrm{n}^2} - \frac{L^2{}_{l1}}{\mathrm{n}^2\ L_P} \end{aligned} \tag{14}$$

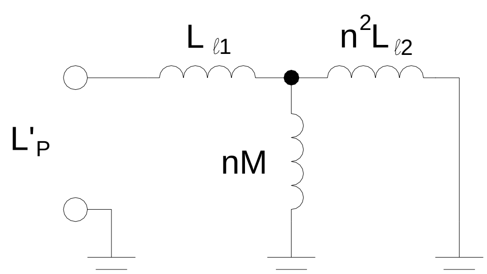

Fig. 6 - Equivalent Circuit for Measuring L'_P

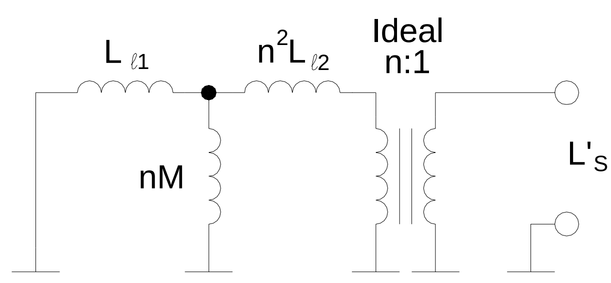

Fig. 7 - Equivalent Circuit for Measuring L'_S

Now, by way of Eq. 10 and Eq. 12, we obtain the following relationship for the secondary leakage inductance L_{l2}:

$$L_{l2} = L_S - \frac{L_P - L_{l1}}{n^2} \qquad (15)$$

Substituting Eq. 15 into Eq. 14, we now arrive at:

$$L'_S = L_S - \frac{L_P - L_{l1}}{n^2} + \frac{(L_P - L_{l1}) L_{l1}}{n^2 L_P} =$$
$$= L_S - \frac{L_P}{n^2} + \frac{2 L_{l1}}{n^2} - \frac{L^2_{l1}}{n^2 L_P}$$
$$= L_{l2} + \frac{L_{l1}}{n^2} - \frac{L^2_{l1}}{n^2 L_P} \qquad (16)$$

which can be arranged to form the following quadratic equation:

$$0 = \left(L_S - L'_S - \frac{L_P}{n^2} \right) +$$
$$+ \left(\frac{2}{n^2} \right) L_{l1} - \left(\frac{1}{n^2 L_P} \right) L^2_{l1} \qquad (17)$$

which can be easily solved for determining the value of the primary leakage inductance L_{l1} and subsequently the primary winding inductance L_1.

In a similar fashion, we now refer to Fig. 6 and recognize that the value of L'_P can be determined by way of:

$$L'_P = L_{l1} + \frac{n^2 M L_{l2}}{M + n L_{l2}} \qquad (18)$$

Substituting Eq. 10 for M in Eq. 18, we obtain:

$$L'_P = L_{l1} + \frac{n (L_S - L_{l2}) n^2 L_{l2}}{n (L_S - L_{l2}) + n L_{l2}} =$$
$$= L_{l1} + \frac{n (L_S - L_{l2}) n^2 L_{l2}}{n L_S} =$$
$$= L_{l1} + \frac{(L_S - L_{l2}) n^2 L_{l2}}{L_S} =$$
$$= L_{l1} + n^2 L_{l2} - \frac{n^2 L^2_{l2}}{L_S} \qquad (19)$$

Rearranging Eq. 12, we obtain the following relationship for the primary leakage inductance L_{l2}:

$$L_{l1} = L_P - n^2 (L_S - L_{l2}) \qquad (20)$$

Substituting Eq. 20 into Eq. 19, we now arrive at:

$$L'_P = L_P - n^2 (L_S - L_{l2}) +$$
$$+ n^2 L_{l2} - \frac{n^2 L^2_{l2}}{L_S} =$$
$$= L_P - n^2 L_S + 2 n^2 L_{l2} - \frac{n^2 L^2_{l2}}{L_S} \qquad (21)$$

which can be arranged to form the following quadratic equation:

$$0 = (L_P - L'_P - n^2 L_S) +$$
$$+ (2 n^2) L_{l2} - \left(\frac{n^2}{L_S} \right) L^2_{l2} \qquad (22)$$

which can be easily solved for determining the value of the secondary leakage inductance L_{l2} and subsequently the secondary winding inductance L_2.

Determining the Model Capacitances

With the transformer model inductances taken care of, we can now turn to the model capacitances, beginning with the interwinding capacitance C_{12}. Referring to Fig. 8, this ca-

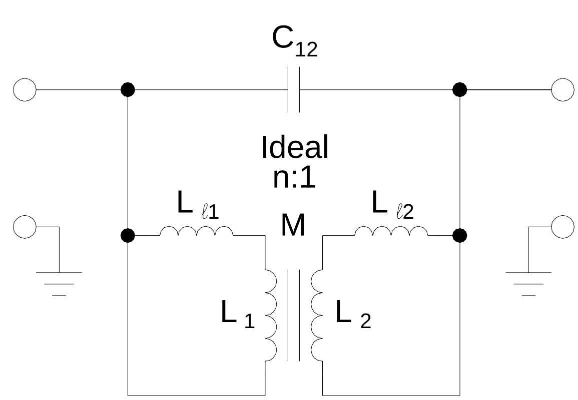

Fig. 8 - Circuit for Measuring C_{12}

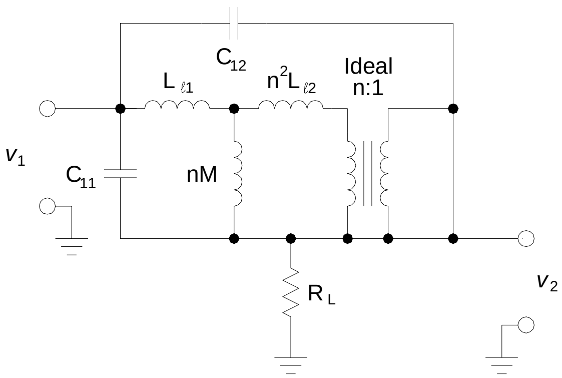

Fig. 9 - Circuit for Measuring C_{EQ1}

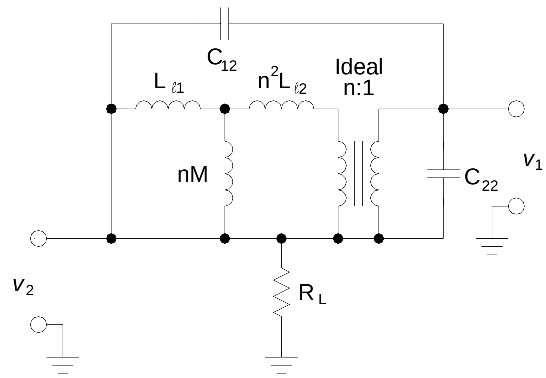

Fig. 11 - Circuit for Measuring C_{EQ2}

pacitance is very easily measured by way of shorting the primary and secondary terminals of the transformer and simply measuring the capacitance (9). Make very careful note here that there is no connection between the transformer terminals and ground.

Measurement of the winding self capacitances C_{11} and C_{22} is a bit more demanding. Referring to Fig. 9, the measurement of the primary intrawinding capacitance C_{11} begins by shorting the secondary terminals of the transformer. Next, the cold end of the primary winding and the shorted secondary winding are connected to a load resistance R_L, very likely the input resistance of a scalar network analyzer. A signal generator supplying a signal voltage V_1 is then connected between the top end of the primary winding and ground. The output voltage V_2 is then monitored as the frequency of the generator is adjusted until a null is reached at a frequency f_{11}.

Referring now to the equivalent circuit of this measurement in Fig. 10, the null frequency f_{11} is the parallel resonant frequency of the two circuit elements C_{EQ1} and L_{EQ1} ,

$$C_{EQ1} = \frac{1}{L_{EQ1}\left(2\pi\, f_{11}\right)^2} \tag{23}$$

where

$$L_{EQ1} = L'_P = L_{l1} + \frac{\mathrm{n}^2 M\, L_{l2}}{M + \mathrm{n}\, L_{l2}} \tag{24}$$

The primary intrawinding capacitance C_{11} is then determined by:

$$C_{11} = C_{EQ1} - C_{12} \tag{25}$$

Referring now to Fig. 11, the measurement of the secondary intrawinding capacitance C_{22} is identical, begining by shorting the

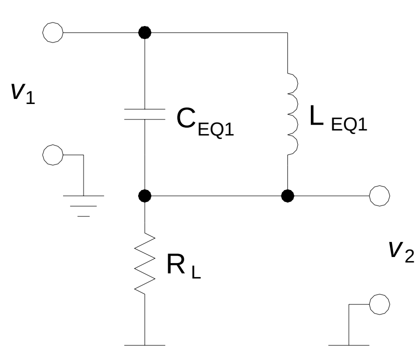

Fig. 10 - Equivalent Circuit for Measuring C_{EQ1}

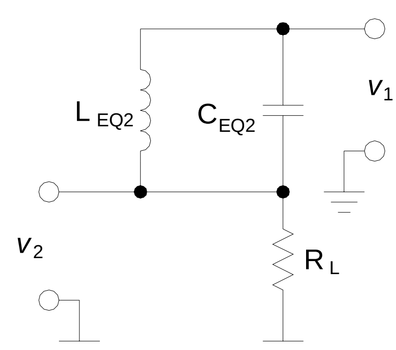

Fig. 12 - Equivalent Circuit for Measuring C_{EQ2}

primary terminals of the transformer. Next, the cold end of the secondary winding and the shorted primary winding are connected to the load resistance R_L. The signal generator supplies a signal voltage V_1 between the top end of the secondary winding and ground. The output voltage V_2 is then monitored as the frequency of the generator is adjusted until a null is reached at a frequency f_{22}.

Referring now to the equivalent circuit of this measurement in Fig. 12, the null frequency f_{22} is the parallel resonant frequency of the two circuit elements C_{EQ2} and L_{EQ2} ,

$$C_{EQ2} = \frac{1}{L_{EQ2}\,(2\pi\ f_{22})^2} \qquad (26)$$

where

$$L_{EQ2} = L_S' = L_{l2} + \frac{M\ L_{l1}}{\mathrm{n}^2 M + \mathrm{n}\, L_{l1}} \qquad (27)$$

The secondary intrawinding capacitance C_{22} is then determined by

$$C_{22} = C_{EQ2} - C_{12} \qquad (28)$$

Closing Remarks

The determination of the inductive and capacitve elements of the lossless wideband transformer model have been shown to be easily determined by way of a series of simple measurements and calculations. This approach to evaluating wideband transformer model elements has been found to yield satisfactory results over a wide range of frequencies and has proven to be a highly usable tool for high frequency circuit design and simulation.

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